Review: Significant Figures, Error and Uncertainty

Propagating Error: Rules

1.Addition and Subtraction: Add absolute Uncertainties

$$(A \pm \Delta A) \pm (B \pm \Delta B) = (A \pm B) \pm (\Delta A + \Delta B)$$

2. Multiplication and Division

For example

$$(A \pm \Delta A) \times (B \pm \Delta B) = C \pm \Delta C$$

Then

$$C = A \times B$$

a) First find the relative uncertainty of each factor, or the percentage relative uncertainty, %:

$$Relative.Uncertainty = \frac{\Delta A}{A}$$

Or it may be expressed as a percentage

$$\% = \frac{\Delta A}{A} \times 100\%$$

b) Add the relative Uncertainties to give the relative uncertainty of C

$$\frac{\Delta C}{C} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

c) Convert the Relative Uncertainty of C back to absolute uncertainty (by multiplying by C)

$$\Delta C = (\frac{\Delta A}{A} + \frac{\Delta B}{B}) \times C$$

3.Multiplying or Dividing by a Pure Number

Multiply or divide the uncertainty by that number.

E.g.

$$(4.35 \pm 0.05) \times 10 = 43.5 \pm 0.5$$

Note: ONLY report uncertainties to 1 significant digit. Very important!

Examples: Right: 153.0 ±0.8 Wrong: 220 ± 0.1 Wrong: 147 ± 10
Right: 150. ±4 Wrong: 220 ± 2.5 Wrong: 556.854±0.01
Also (1.50 ±0.04) x 10^2

Practice Exercises

1. Change the following absolute uncertainties to relative (%) uncertainties.

a) 36.02 ± 0.02 m

b) 4.1 ± 0.1 cm

c) 17.83 ± 0.01g

d) 2.45 ± 0.01 kg

e) 123.95 ± 0.05 L

2. Convert the following percentage error to absolute error.

a) 43.00 ± 0.2%

b) 8.32 ±3%

c) 1.098 ± 0.1%

d) 15.5 ± 5

Significant Figures, Error and Uncertainty

1. Propagate the error in the following calculations:

a) $12.2 \pm 0.3 + 15.5 \pm 0.5$

b) $9.45 \pm 0.01 - 8.34 \pm 0.1$

c) $(5.5 \pm 0.5) \times (12.8 \pm 0.5)$

d) $(2.1 \pm 0.1) \times (1.5 \pm 0.1) \times (8.2 \pm 0.05)$

e) $(21 \pm 3) \times (12.5 \pm 0.7)$

f) $(100 \pm 1) \times (500 \pm 1)$

Problems:

1) Calculate the density given the following:

   Mass = 1.23 ± 0.01g V = 0.56 ± 0.05 cm^3

Answer: 2.2 ± 0.2 g/cm^3

2) Calculate the density with the following dimensions:

L= 3.42 ± 0.05 cm, W = 1.2 ± 0.2 cm, H = 54.85 ± 0.02 cm, mass =153 ± 2 g

Answer: 0.7 ± 0.1 g/cm^3

3) Calculate the concentration in g/L of a sodium chloride solution using the following data:

   mass of beaker + NaCl = 241.85 ± 0.01 g,

   mass of empty beaker = 159.23 ± 0.01 g,

   volume of water = 250. ±1 mL

Answer: 330. ± 1g/L

More practice problems, very exciting!

1. a) Determine the volume:

   L = 3.50 ± 0.01cm, H = 3.53 ± 0.01cm, W = 3.55 ± 0.01cm

   Answer: 43.9 ± 0.9%

   b) If the mass of the cube in 1(a) above is 125.52 ± 0.01g, what is the density?

   Answer: 2.86±0.03g/cm^3

2. Calculate the density of an object which has the following measurements:

   Mass = 20.28 ± 0.02 g, volume = 24.01 ± 0.01 mL

   Answer: 0.845 ± 0.001 g/mL

3. Calculate the speed of an object if the distance travelled is 200.00±0.01m in 21.99 ± 0.01 s. (Recall: distance = speed x time)

   Answer: 9.10 ± 0.05 m/s

4. A car travels at a speed of 100. ± 5 km/hr for 3.2 ± 0.1hours. What is the distance travelled?

   Answer: 320 ± 30km

5. Calculate the quantity of heat absorbed, Q, by $250 \pm 5\ mL$ of water, having an initial temperature of $32.0 \pm 0.5℃$ and a final temperature of $89.5 \pm 0.5℃$.

Use specific heat capacity of water, $c = 4.18 \frac{J}{g℃}$.

Use $Q = mc\Delta T$ $\quad (\Delta T = T_f - T_i)$

m = mass

c = specific heat capacity

(Answer: $\boldsymbol{Q = 60.1 \pm 2.2\ kJ}$)

6. Calculate the number of moles, n, of sodium hydroxide, NaOH, given the following data:

Mass of paper + mass of sodium hydroxide $= 2.51 \pm 0.01\ g$

Mass of paper $= 0.06 \pm 0.01\ g$

Molar mass of NaOH $= 40.0\ g\ mol^{-1}$

Use: $n^o\ moles, n = \frac{mass}{molar\ mass}$

(Answer: $\boldsymbol{n^o\ moles = 0.613 \pm 0.02\ moles}$)

Division Problems: (Very Exciting!)

1. Complete the following, determining the appropriate uncertainty:
a) (12.02 ± 0.08 cm) ÷ (16 s ± 8 %)

b) (3.5 cm ± 10 %) × (2.70 ± 0.05 cm) ÷ (16 s ± 8 %)

c) $(12.02 \pm 0.08 \text{ cm})^2 \div (3.5 \text{ cm} \pm 10\ \%)$

d) $(12.02 \pm 0.08 \text{ cm})^2 + (3.5 \text{ cm} \pm 10\ \%) \times (2.70 \pm 0.05 \text{ cm})$

e) [(3.5 cm ± 10%) + (2.70 ± 0.05 cm)] / (16 s ± 8%)

f) $4p^2 / (0.034 \pm 0.004 \text{ cm/s}^2)$

2. Determine the perimeter and area of a rectangle of length 9.2 ± 0.05 cm and width 4.33 ±0.01 cm.

3. A block of wood measures (12.0 ± 0.5 cm) by (25.1 ± 0.1 cm) by (62 ± 1 cm). If it has a mass of (9.60 kg ± 5%), what is its density?

Answers

1. a) 0.75 cm/s ± 9% = 0.75 ± 0.07 cm/s
b) 0.59 cm2/s ± 20% = 0.59 ± 0.12 cm2/s = 0.6 ± 0.1 cm2/s
c) 41 cm^2 ± 10% = 41 ± 4 cm^2
d) 154 ± 3 cm^2
e) either 0.39 cm/s ± 15% or 0.39 cm/s ± 20%, and if you convert to absolute, you'll get either 0.39 ± 0.06 cm/s or 0.39 ± 0.08 cm/s, depending on when you did your rounding.
f) 1161.13 cm/s^2 ± 10 % = 1200 ± 100 cm/s^2

2. P = 27.1 ± 0.1 cm
A = 39.8 cm^2 ± 0.8 % = 39.8 ± 0.3 cm^2 (use precision of uncertainty to decide on precision of answer)

3. m = 9.60 kg ±5%
w = 12.0±0.5 cm = 0.120 m ±4.2%
l = 25.1±0.1 cm = 0.251 m ±0.40%
h = 62±1 cm = 0.62 m ±1.6%

V = lwh
D = m/v = m/lwh
D = (9.60 kg ±5%)/(0.251 m ±0.40%)(0.120 m ±4.2%)(0.62 m ±1.6%)
D = 514 kg/m^3 ±11.2%
D = 514 ±57.5 kg/m^3
D = 510 ±60 kg/m^3